A NEW EVALUATION CRITERION FOR POINT CORRESPONDENCES IN STEREO IMAGES

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ABSTRACT

In this paper, we present a new criterion to evaluate point correspondences within a stereo setup. Many applications such as stereo matching, triangulation, lens distortion correction, and camera calibration require an evaluation criterion, indicating how well point correspondences fit to the epipolar geometry. The common criterion here is the epipolar distance. Since the epipolar geometry is often derived from noisy and partially corrupted data, an uncertainty regarding the estimation of the epipolar distance arises. However, the uncertainty of the epipolar geometry, in the shape of the covariance matrix of an epipolar line, provides additional information, and our approach utilizes this information for a new distance measure. The basic idea behind our criterion is to determine the most probable epipolar geometry that explains the point correspondence in the two views. Furthermore, we show that using Lagrange multipliers, this constrained minimization problem can be reduced to solving a set of three linear equations.

1. INTRODUCTION

For a given point in one image, a so-called epipolar line in the second image can be derived, and a corresponding point should be located on that line. Stereo correspondence algorithms may benefit from this fact in the sense that a correspondence search can be restricted to a small area around an epipolar line, or that outliers may be detected. In [8], a unified correspondence framework was introduced that covers the homographic and epipolar extremes and limits the search region for point correspondences. Many different distance measures, summarized in [3], evaluate how well a pair of points satisfies the epipolar geometry. All of these measures treat the epipolar distance equally throughout the image plane and none of these measures takes into account the uncertainty of the epipolar geometry. However, in estimation theory it is well known that predictions of an estimator near the mass center of the data set, which was originally used to compute the parameters of the estimator, are more accurate compared to predictions at the border of the data set [6]. Furthermore, since the covariance matrix of an epipolar line depends on the position of the according point correspondence, the reliability of the epipolar geometry is not homogeneously distributed, see Fig. 1 and Fig. 2. Recently, in [1] a method for calculating a probability density function for point correspondences was introduced, where for each pixel the summarized probability of all possible epipolar lines going through this pixel is computed. This implies that point correspondences, having a disparity similar to the mean disparity of the data set, become more likely than uncommon disparities. In this paper, we present a novel distance measure that takes into account the uncertainty of the epipolar geometry in a sound way. We show that, using Lagrange multipliers, the constrained minimization problem can be reduced to solving a set of three linear equations. We show the benefits of our criterion in an outlier removal task.

2. PROBABILISTIC EPIPOLAR GEOMETRY

In practice, due to noise and outliers, only an approximation of the true epipolar geometry may be estimated. Common estimation approaches like the normalized 8-point algorithm [3] and [9], as well as complex iterative estimation approaches like [7], which take into account the heteroscedasticity of the underlying problem, have been developed. A good overview on recent methods for computing the fundamental matrix is given in [5]. In the context that point correspondences are only perturbed by Gaussian noise, these methods can be regarded as nearly optimal. As the point correspondences are also interstratified by outliers, these methods have a reduced accuracy and a decision has to be made for each correspondence, whether it is an outlier [10].

2.1. Epipolar line and epipolar envelope

For a given point $\mathbf{x}$ in the first image and a fundamental matrix $\mathbf{F}$, the corresponding epipolar line $\mathbf{l}$ in homogeneous coordinates is given by

$$\mathbf{l} = \mathsf{F}\mathbf{x}. \quad (1)$$

From the covariance matrix Σ_{F} of the fundamental matrix $\mathbf{F}$, which may be directly estimated by a method described in [2],

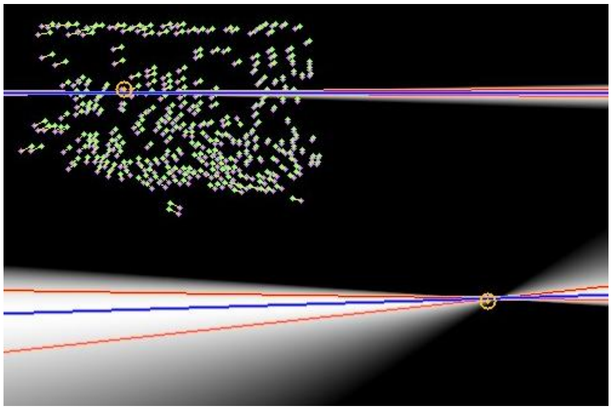

Fig. 1. Uncertainty of the epipolar geometry: Point correspondences composing the data set, used to estimate the epipolar geometry and its uncertainty (Upper left corner). Epipolar band (k^2 values in gray scale), epipolar envelope (green, $\alpha = 0.95$ interval), and most probable epipolar line (blue) for a point near the mass center of the data set and outside of it (yellow). The narrowness of the epipolar band indicates the accuracy of the epipolar line estimation. Hence, epipolar lines for point correspondences located near the mass center of the data set are estimated more accurate compared to correspondences at the border.

we can obtain the covariance matrix of the epipolar line by

$$\Sigma_{\mathbf{l}} = \mathsf{J}\Sigma_F\mathsf{J}^\mathsf{T}, \tag{2}$$

where J is the Jacobian of the mapping

$$\mathbf{l} = (\mathsf{F}\mathbf{x})/\|\mathsf{F}\mathbf{x}\|. \tag{3}$$

The Mahalanobis distance k^2 between an arbitrary epipolar line $\mathbf{u}$ and the estimated epipolar line $\mathbf{l}$ is given by

$$(\mathbf{l} - \mathbf{u})^\mathsf{T} \Sigma_{\mathbf{l}}^+ (\mathbf{l} - \mathbf{u}) = k^2. \tag{4}$$

Thus, we have an approximation, how well an arbitrary line $\mathbf{u}$ matches with the knowledge acquired so far about the epipolar geometry and its uncertainty. For any given value of k^2, we can compute an envelope of epipolar lines containing all possible epipolar lines having a value less or equal to k^2. The envelope is described by a conic C defined in homogeneous coordinates by

$$\mathsf{C} = \mathbf{l}\mathbf{l}^\mathsf{T} - k^2\Sigma_{\mathbf{l}}. \tag{5}$$

With the assumption that the elements of $\mathbf{l}$ follow a normal distribution, k^2 must follow a cumulative χ^2_2 distribution. In addition, the probability that the true epipolar line is located within this envelope can be associated to the k^2-value, where $\alpha = F_2(k^2)$ can be regarded as the probability to find $\mathbf{x}'$ within C.

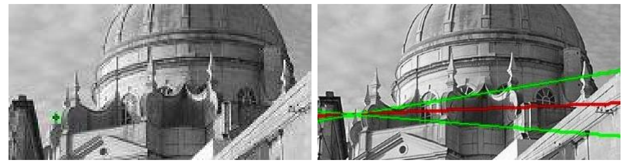

Fig. 2. Envelope of epipolar lines: Point in the first image used for computation(green, $k^2 = 5.9915$, $\alpha = 0.95$ interval). Most likely epipolar line (red) and epipolar envelope (green) in the second image.

3. NEW DISTANCE MEASURE FOR POINT CORRESPONDENCE EVALUATION

3.1. Basic idea

The basic idea for the new criterion is to invert the problem of finding an epipolar band for a given likelihood (i.e. a given k^2 or respectively a probability α). For a given point $\mathbf{x}$ in the first view and a corresponding point $\mathbf{x}'$ in the second view, we want to find the conic with minimal k^2 comprising the point $\mathbf{x}'$. In other terms, we are retrieving the smallest value k^2 in equation 5 that provides a hyperbola passing through $\mathbf{x}'$.

3.2. General Problem Statement

The point $\mathbf{x}'$ belongs to the conic C if the following equation is verified

$$\mathbf{x}'^\mathsf{T}\mathsf{C}\mathbf{x}' = 0, \tag{6}$$

where C is given by:

$$\mathsf{C} = \mathbf{u}\mathbf{u}^\mathsf{T} - k^2\Sigma_{\mathbf{l}} \tag{7}$$

It is not possible to retrieve directly the corresponding value k^2 from a point $\mathbf{x}'$ using the equations above, but we found a closed-form solution to the problem, providing an exact result.

3.3. Closed-form solution

Let us assume that the confidence, that any point $\mathbf{x}'$ in the second image, is corresponding to a point $\mathbf{x}$ in the first view is in relation with the probability of the epipolar geometry that would explain the correspondence pair $\mathbf{x} \leftrightarrow \mathbf{x}'$ and having at the same time the highest probability. In other terms, for a potential point correspondence $\mathbf{x} \leftrightarrow \mathbf{x}'$, we retrieve the epipolar line $\mathbf{u}$ passing through $\mathbf{x}'$ having maximal probability, i.e. minimal k^2 regarding equation 4. This assumption differs from the assumption made in [1], and we obtain a different criterion, which is more suitable for our purpose.

If we denote with $\mathbf{u}$ the unknown epipolar line and with $\mathbf{l}$ the estimated line in the second image, the constrained minimization problem can be stated as follows:

$$\begin{cases} \min f(a,b,c) = (\mathbf{u}-\mathbf{l})^{\mathsf{T}} \Sigma_{\mathbf{u}}^{+} (\mathbf{u}-\mathbf{l}) \\ g(a,b,c) = \mathbf{x'}^{\mathsf{T}}\mathbf{u} = 0 \end{cases}, \quad (8)$$

with $\mathbf{x}' = (x, y, 1)$, $\mathbf{l} = (l_1, l_2, l_3)^{\mathsf{T}}$, $\mathbf{u} = (a, b, c)^{\mathsf{T}}$, and the inverse covariancematrix $\Sigma_{\mathbf{l}}^{+}$ derived by an SVD which is still a symmetric matrix of the form

$$\Sigma_{\mathbf{l}}^{+} = \begin{pmatrix} \sigma_{11}^2 & \sigma_{12}^2 & \sigma_{13}^2 \\ \sigma_{12}^2 & \sigma_{22}^2 & \sigma_{23}^2 \\ \sigma_{13}^2 & \sigma_{23}^2 & \sigma_{33}^2 \end{pmatrix}. \quad (9)$$

For solving the minimization problem, we choose the Lagrange multipliers method to obtain an exact solution by solving the set of equations:

$$\begin{array}{rcl} \nabla f(a,b,c) & = & \lambda \nabla g(a,b,c) \\ g(a,b,c) & = & 0 \end{array}. \quad (10)$$

After expanding $f(a, b, c)$ and computing its derivatives, we have:

$$\begin{array}{l} \frac{\partial f}{\partial a} = 2\left(a\sigma_{11}^2 + b\sigma_{12}^2 + c\sigma_{13}^2 - l_1\sigma_{11}^2 - l_2\sigma_{12}^2 - l_3\sigma_{13}^2\right) \\ \frac{\partial f}{\partial b} = 2\left(a\sigma_{12}^2 + b\sigma_{22}^2 + c\sigma_{23}^2 - l_1\sigma_{12}^2 - l_2\sigma_{22}^2 - l_3\sigma_{23}^2\right) \\ \frac{\partial f}{\partial c} = 2\left(a\sigma_{13}^2 + b\sigma_{23}^2 + c\sigma_{33}^2 - l_1\sigma_{13}^2 - l_2\sigma_{23}^2 - l_3\sigma_{33}^2\right) \end{array}. \quad (11)$$

For $g(a, b, c)$ we get the following derivatives:

$$\partial g/\partial a = x \quad \partial g/\partial b = y \quad \partial g/\partial c = 1 . \quad (12)$$

Thus we get for

$$\begin{array}{lcl} \lambda\partial g/\partial a = \partial f/\partial a & \Leftrightarrow & \lambda = \frac{1}{x}\partial f/\partial a \\ \lambda\partial g/\partial b = \partial f/\partial b & \Leftrightarrow & \lambda = \frac{1}{y}\partial f/\partial b \\ \lambda\partial g/\partial c = \partial f/\partial c & \Leftrightarrow & \lambda = \partial f/\partial c \end{array} \quad (13)$$

As we are solving a problem with three unknown variables, three equations are sufficient. Using the relations from equation 13 and the constraint that the point is located on the line $\mathbf{u}$, we obtain the set of equations:

$$\begin{array}{rcl} \partial f/\partial a & = & x\partial f/\partial c \\ \partial f/\partial b & = & y\partial f/\partial c \\ ax + by + c & = & 0 \end{array}. \quad (14)$$

Expanding equation 14 we get the linear equation system

$$\begin{pmatrix} & \mathbf{A} & \\ x & y & 1 \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \mathbf{A} \\ \mathbf{0} \end{pmatrix} \cdot \mathbf{l}, \quad (15)$$

where $\mathbf{A}$ is a 2×3 matrix:

$$\mathbf{A} = \begin{pmatrix} \sigma_{11}^2 - x\sigma_{13}^2 & \sigma_{12}^2 - x\sigma_{23}^2 & \sigma_{13}^2 - x\sigma_{33}^2 \\ \sigma_{12}^2 - y\sigma_{13}^2 & \sigma_{22}^2 - y\sigma_{23}^2 & \sigma_{23}^2 - y\sigma_{33}^2 \end{pmatrix}. \quad (16)$$

This set of equations can be easily computed and the solution vector $(a, b, c)^{\mathsf{T}}$ is the line $\mathbf{u}$ passing through $\mathbf{x}'$ and having minimal k^2 in terms of equation 8. Finally, we obtain the corresponding value k^2 from 4 that induces an epipolar band delimited by a hyperbola passing through $\mathbf{x}'$. The value k^2 is of special interest. This is the new distance measure we introduce in this paper.

4. APPLICATION TO OUTLIER REMOVAL

Although a thorough analysis of the influence of the criterion on all possible applications is beyond the scope of this paper, we investigated a simple algorithm for fundamental matrix computation that was enhanced by using our measurement, in order to give an example of an application.

4.1. Algorithm

To compare our new criterion with the epipolar distance, we use an iterative version of the normalized 8-point algorithm. In each iteration, the minimal k^2 distance or respectively the epipolar distance of the point correspondences to their epipolar lines are computed. If a correspondence exceeds a threshold based on the mean distance of all point correspondences, it is classified as an outlier and removed from further usage. After some iterations or if no significant outliers are detected, the process stops and a final fundamental matrix is computed. In order to distinguish between the two methods, the one which uses our new distance measure is referred to as 'Statistic', while the method comprising the Euclidean epipolar distance is called 'Euclidean'. Both methods are identical except for the distance measures used to eliminate outliers. The resultant fundamental matrices have to be compared in a sound way. Since we replaced the epipolar distance, the 'Statistic' method does not optimize the Sampson distance [3] any more. Hence, common quality measurements like the RMS, or the mean epipolar distance are not practicable, even though our method well recognized by them, as we will see. Instead, we used a simulation environment, where a true fundamental matrix as well as the intrinsic camera parameters are known. This allows to evaluate the similarity between the estimated fundamental matrices and the ground truth. This is accomplished by separating the $\mathbf{F}$-matrices into rotation matrix and translation components, so that the angles between the rotations and translation can be compared. (see [3] for decomposition details). Nevertheless, we also computed results showing RMS-error and mean epipolar distance values.

4.2. Results

In order to create a reasonably appropriate scene, we created several 3D-world setups at random so that 70 percent of the point correspondences are located on a wall like structure and 30 percent on a floor-like plane. We mapped the 3D points into the camera planes, where Gaussian noise with $\sigma = 0.5$ and 10 percent outliers were added. From those point correspondences the fundamental matrix was estimated by the 'Statistic' and 'Euclidean' method. In addition, we computed the fundamental matrix using the RANSAC method from the OpenCV [4] and the HEIV method from [7] and compared their results as well. We show the results for varying the total number of point correspondences, since it affected the result more than the variation of the structure in the scene. Figure

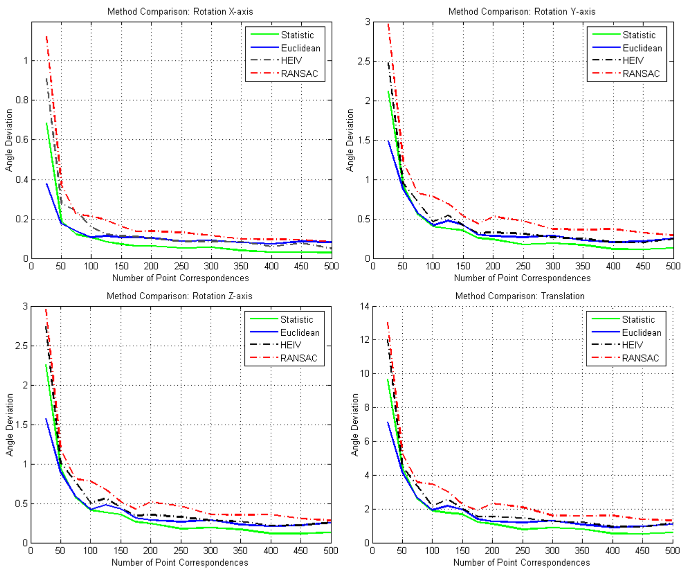

Fig. 3. Fundamental matrix decomposition: Angle deviation in degree from ground truth for the R_x, R_y, R_z rotation and translation components.

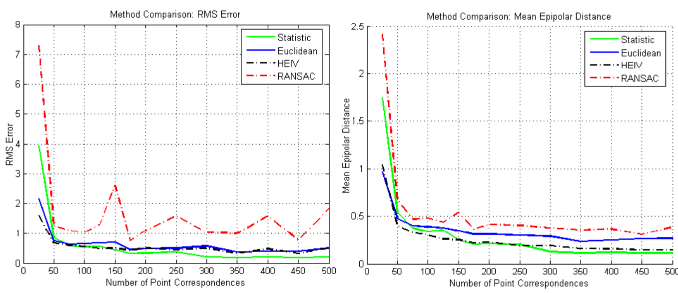

Fig. 4. RMS-error and mean epipolar distance computed for each method.

3 shows the angle deviation compared to the ground truth for all four methods regarding the rotation components and the translation. Since all methods more or less depend on statistics or contain a random generator, for each data point in the diagrams, we computed 100 trials and averaged them. They show that the 'Statistic' method, which comprises our new criterion, performs very well as soon as sufficient ($n > 100$) point correspondences are available. The reason for this is that the computation of the covariance matrix requires the estimation of additional parameters, which is less accurate for a low number of point correspondences. In fig. 4 the RMS-error as well as the mean epipolar distance of all methods is displayed. The 'Statistic' method performs well in this context. The direct comparison to the 'Euclidean' method shows that, especially for a higher number of point correspondences, our new distance measure leads to a better outlier removal. An important observation is that the RMS-error and the mean epipolar distance are not always consistent with the ground truth comparison, which was assumed above and is an additional justification for a new measurement criterion. E.g. the HEIV method is rated better in most cases than the 'Euclidean' method by RMS and mean epipolar distance, but the comparison with the ground truth reveals that this evidence can not be supported. Also the relative distances between the other methods among each other behave different.

5. CONCLUSION

We have presented a novel criterion to evaluate point correspondences in stereo images. Our main contribution is to show that the uncertainty of the fundamental matrix can be integrated into the distance measure in a sound way. We showed that it is possible to find the minimal k^2 value without exceedingly additional computational cost. We demonstrated the benefits of our criterion for the computation of the fundamental matrix by enhancing the outlier detection.

6. REFERENCES

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